Kiril Datchev
MA 425/525
Fall 2023

Homework 8

Due November 29th on paper at the beginning of class. Justify your answers. Please let me know if you have a question or find a mistake. The book is Fisher's `https://archive.org/details/complex-variables-2ed-dover-1999-fisher/page/n23/mode/2up`.

1. Let $f(z) = \sum_{n=-1}^{1} z^n$. Let $F(z)$ be the antiderivative of $f(z)$ on $\{z \colon z \neq 0 \text{ and } \mathrm{Arg} z \neq -\pi/4\}$ such that $F(2) = 3$. What is $F(i)$?
2. How does the answer to #1 change if $-\pi/4$ is replaced by $\pi/4$?
3. Show that if f and g are any complex-valued functions such that $|f(z) + g(z)| < |f(z)|$ for all z in some set Γ, then neither f nor g can have any zeroes on Γ.
4. (a) Find the velocity function $v(z)$ for Example 1 from page 4 of Eremenko's `https://www.math.purdue.edu/~eremenko/dvi/airplanes.pdf`.
   (b) Find constants a, b, and c such that $v(e^{i\theta}) = 1 + ae^{bi\theta} = ce^{i\theta} \sin\theta$.
   (c) What property of the constant c above implies that $v(e^{i\theta})$ is perpendicular to $e^{i\theta}$?
   (d) Sketch the vector field corresponding to $v(z)$ on the circle $|z| = 1$ (as in Section 2.1.1 of Fisher), and mark the points where $|v(e^{i\theta})|$, $\mathrm{Re}\, v(e^{i\theta})$, and $\mathrm{Im}\, v(e^{i\theta})$ achieve their maximum and minimum values.

Hints: For #3, show that if $f(p) = 0$ or $g(p) = 0$, then $|f(p) + g(p)| \geq |f(p)|$. For #4a, substitute $f(z)$ into equation (3) on page 1 of Eremenko. For #4c use page 8 of Fisher.