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MA 425/525
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Homework 8

Due November 20th on paper at the beginning of class. Justify your answers. Please let me know if you have a question or find a mistake. The book is `https://archive.org/details/complex-variables-2ed-dover-1999-fisher/page/n23/mode/2up`.

Do 3.3.4b and 3.3.5ac from page 204, and 3.4.7d from page 218.

Also do the following:

1. Let $f(z) = \sum_{n=-1}^{1} z^n$. Let $F(z)$ be the antiderivative of $f(z)$ on $\{z\colon z \neq 0 \text{ and } \operatorname{Arg} z \neq -\pi/4\}$ such that $F(2) = 3$. What is $F(i)$?
2. How does the answer to the previous problem change if $-\pi/4$ is replaced by $\pi/4$?
3. Let U be the upper half plane $\{z\colon 0 < \operatorname{Arg} z < \pi\}$. Find a one-to-one analytic function f from U onto U such that $f(1+i) = -3+2i$.
4. Let Q be the first quadrant. Find a one-to-one analytic function f from Q onto Q such that $f(3+4i) = 4+3i$.
5. Describe and sketch the image of $\{z\colon \operatorname{Re} z > 0, 0 < \operatorname{Im} z < 1\}$ under the mapping $f(z) = \frac{z-1}{z+1}$.

Hints: For 3.3.4b, write $T(z) = a(z-p)/(z-q)$, then find p using the information for which point is sent to 0, then a using the information for where ∞ is going, then last q. For 3.3.5a use $z \mapsto 1/(z-1)$ to take the circle to a line. Then you can map one line to another using a linear map. For #1 and #2, pay attention to the branch of the logarithm used in F. For #3, use a linear function. For #4, use the mapping $z \mapsto z^2$ to convert the given problem to a problem on U, like in #3. For #5, describe and sketch the image of the boundary of the region, using the fact that linear fractional transformations take lines and circles to lines and circles.